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ON PREDICTING PRODUCTION COSTS AND PROBABLE LEARNING RATES FROM R&D INVESTMENTS BY S-CURVE/LEARNING CURVE RELATIONSHIPS

by

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PREFACE

The learning curve, a dynamic rather than a static concept, is a powerful tool of cost forecasting and control. At present, the surface of its potential, outside the aircraft and missile industry, has only been scratched. Voluminous literature exists on the learning curve, unfortunately most of it is limited to the basic ideas of T. P. Wright in 1936. Cochran, recognizing this malady of contemporary learning curve methodology, has introduced several new concepts of learning curve analysis. One of these new concepts is the S-Curve/Learning Curve relationship, developed to determine the cost of the effect of engineering changes in an operating system.

The principal purpose of this paper is the expansion of the S-Curve concept of cost estimating to include research and development costs and their relation to production costs. The premise here is: as research brings about change and development is change, then research and development can be correlated directly with change.

The theme is directed toward the cost estimating defense community and it is hoped that this paper will provide stimulation for further study and use of this dynamic tool, the learning curve.

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1. INTRODUCTION

As a result of the introduction of changes to an operating production system, there are three separate and distinct costs that must be considered, (1) the cost of the effect of the changes, (2) the cost of the changes, and (3) the basic production cost had the changes not occurred.

Cochran[1/] developed an S-curve/log-linear curve relationship for determining one of these costs, the cost of the effect of change in the production phase. This paper expands this concept to include (1) the cost of the change in the production phase, by introducing a third log-linear curve, (2) the cost of the effect of change in the research and development (R&D) phase, and (3) the cost of the change in the R&D phase through a second S-curve and triple log-linear curve relationship. The supposition is:

a. that in a given R&D situation if these two costs are established the base or production cost for the R&D and full scale production phases can be determined.

b. S-curve/log linear curve relationships can be developed to establish en masse for a given configuration the R&D prototype or preproduction model and first unit production costs, full scale production preproduction model and first unit production costs, the R&D production learning rate, the state-of-the-art, and the probable full scale production learning rate.

The paper is written in the general form, the formulae are readily adaptable for the generation of tables similar to those in existence for the basic learning curve.[2] An example is given. The principles of the learning curve concept introduced by Wright and Crawford are not given as literature abounds on this concept.[3] [4] The more important references are noted in the text and listed in the Reference List. The concept is applicable in both the R&D and production phases and is appropriate in any situation that lends itself to normal learning curve methodology.

2. <u>COCHRAN'S S-CURVE CONCEPT</u>

Cochran[1] empirically developed the S-curve depicted in Figure 1 to measure the <u>effect</u> of changes introduced in an existing manufacturing process. To determine the S-curve for a given occurrence, Cochran prescribes six (6) steps. Those steps are:

> Step 1 in the construction is to place the "standard cost" (taken in this example as 100 hours) for unit 1000. In the example shown in Figure 1 this is A, the Standard Cost Point, and has been placed exactly on unit 1000 for simplicity only.
>
> Step 2 is to draw the log-linear learning curve appropriate to the type of work performed. In the example this is assembly; and so a 75% curve is drawn from point A. Since this curve is to be a basic reference point for the cost function, we shall give it the name "characteristic curve".

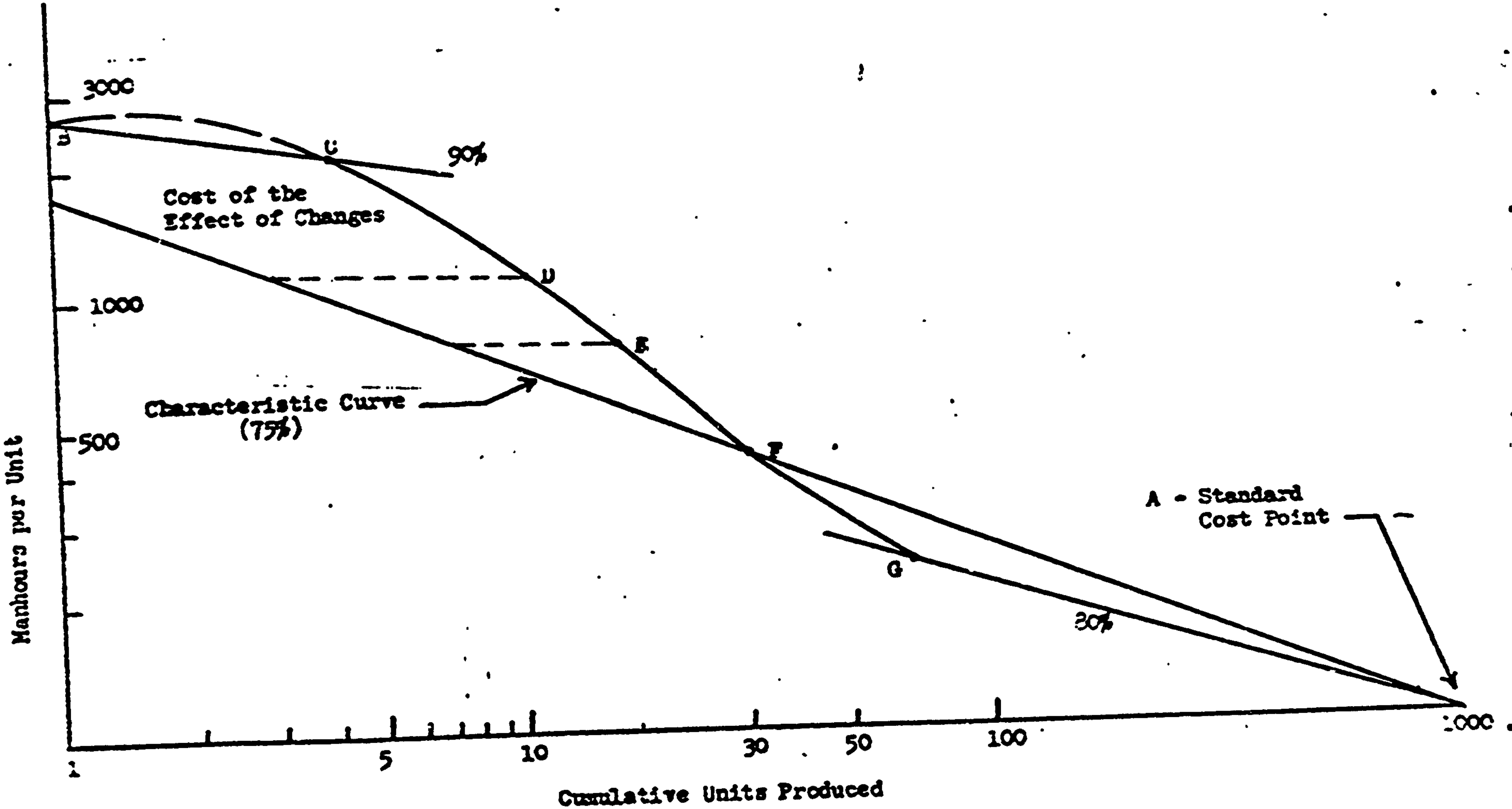


Fig. 1. Cochran's Basic S-curve

Step 3 is to determine the cost of unit one. This is taken as about 50% more than the cost indicated by the characteristic curve. The exact ratio is a matter of judgement, depending upon the newness of the product to company know-how, the degree of pre-planning to be performed and the early impact of engineering changes on tooling and methods.

Step 4 is to mark off several cost values along the S-curve. Point C, the cost as unit 4, is taken at the level indicated by a 90% curve from unit 1. Point D, the cost at unit 10, is taken from the characteristic curve at unit 3, and Point E, is similarly taken from between units 7 and 8. Point F, the cost of unit 30, is set as the spot where the S-curve crosses over the characteristic curve.

Step 5 is to establish Point G. This point is where the S-curve intersects an 80% curve from Point A. It should be moved in or out in direct relationship to Point A: where A is at 1000, G should be at unit 70;where A is at 1500, G should be at 105, etc., holding at roughly 7% of Point A under normal conditions. From Point G on, the S-curve follows the 80% log-linear curve.

Step 6 is to connect all points with a smooth curve.

Further, Cochran states that in the case of cost centers whose characteristic curve would be of a different slope than 75%, reference lines of correspondingly different slopes are appropriate. Proportions to the amount of learning in a learning curve should be used as follows:

	LEARNING CURVE SLOPES		
	ASSEMBLY	FABRICATION	WELDING
Characteristic Line	75%	90%	85%
Point C Line	90%	96%	94%
Point G Line	80%	92%	88%

For assembly, the learning in the characteristic line is 25 points, while that in its Point C line is 10 points and its Point G line 20 points - or 40% and 80%, respectively. Therefore, when the characteristic line is 90% - as for fabrication activities - since there is 10 points of learning, the other two reference points for the S-curve will be 4 points (96%) and 8 points (92%).

The proof in the construction of the S-curve is that <u>the total cost for the S-curve equals the total cost for the characteristic curve</u>, i.e.,

$$\sum_{n=1}^{n=A} S(n) = \sum_{n=1}^{n=A} C(n)$$

where, $S_{(n)}$ = S-curve

$C_{(n)}$ = the log-linear characteristic curve

A = the standard cost point.

For broad application in cost estimating, Cochran's concept is somewhat limited as it (1) evaluates only the cost of the effect of change, (2) requires a matter of judgement in establishing unit one cost of $S_{(n)}$, and (3) utilizes a log-linear curve approach to establish unit four cost of $S_{(n)}$ which becomes inadequate as the ratio between the unit one costs of $S_{(n)}$ and $C_{(n)}$ approaches or becomes greater than 1:1 of $C_{(n)}$.

3. EXPANSION OF THE S-CURVE TO DETERMINE THE BASE PRODUCTION COST

The b... production cost can be determined in the subsequent manner, see Figure 2:

> Step 1 is to construct the S-curve per Cochran's concept
>
> Step 2 is to project back from $S_{(100)}$ a production characteristic curve $Y_{(n)}$ parallel (the same slope) to $C_{(n)}$

This is derived from:

(i) the concept of cost reduction as a result of repetition that is inherent in the log-linear curve

(ii) the cumulative average factor for $S_{(100)}$ equals

$$1 - [(C(100) - Y(100))/Y(100)]$$

In observing Fig. 2, one will note that should a production characteristic curve $Y(n)$ be projected back from $S(70)$ parallel to $C(n)$ the result would be a lower unit one base production cost, and, consequently, a lower total base production cost for units one through seventy than when $Y(n)$ is projected back from $S(100)$. Similarly, if $Y(n)$ is projected back from a unit greater than $S(100)$, say $S(110)$, the result will be a larger unit one and total base production cost for units one through seventy.

If follows that the basic production cost PC is:

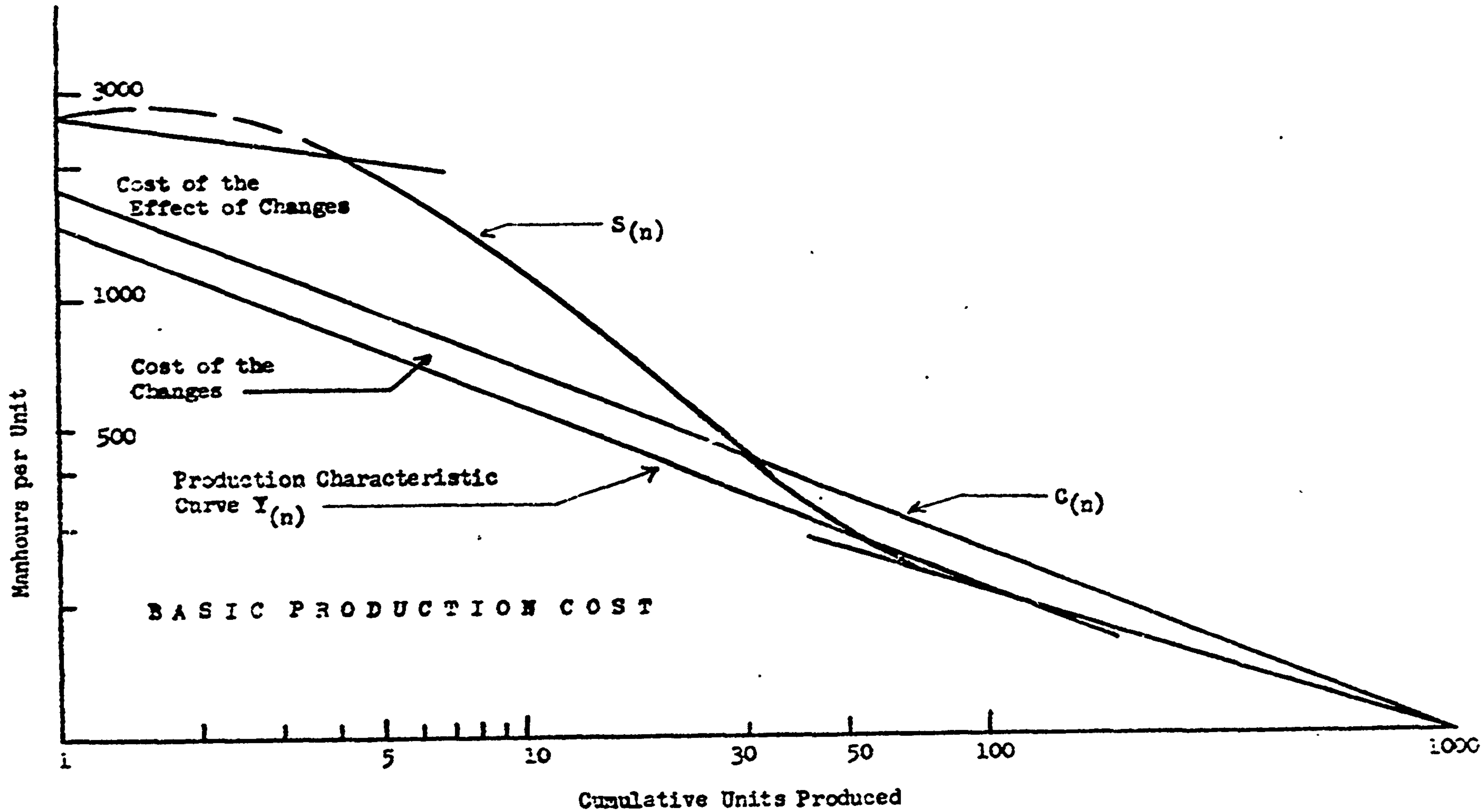


Fig. 2. Modified S-Curve for Determining the Basic Production Cost

$$PC = \sum_{n=1}^{n} Y(n), \text{ where } n \leq 100$$

and the total cost of change TC, i.e., the cost of the effect of change plus the cost of the change is:

$$TC = \sum_{n=1}^{n} S(n), \; n \leq 100$$

4. EXPANSION OF THE S-CURVE TO INCLUDE R&D COSTS

4.1 R&D Prototype and Preproduction Model Costs

From the basic S-curve, let

$S_{(n)} = R\&DS_{(n)}$, representing the R&D prototype and preproduction model costs

$C_{(n)} = R\&DC_{(n)}$, the R&D characteristic curve

$P_{(n)}$ = the expected full scale production curve

$Y_{(n)} = R\&DY_{(n)}$, the R&D production base.

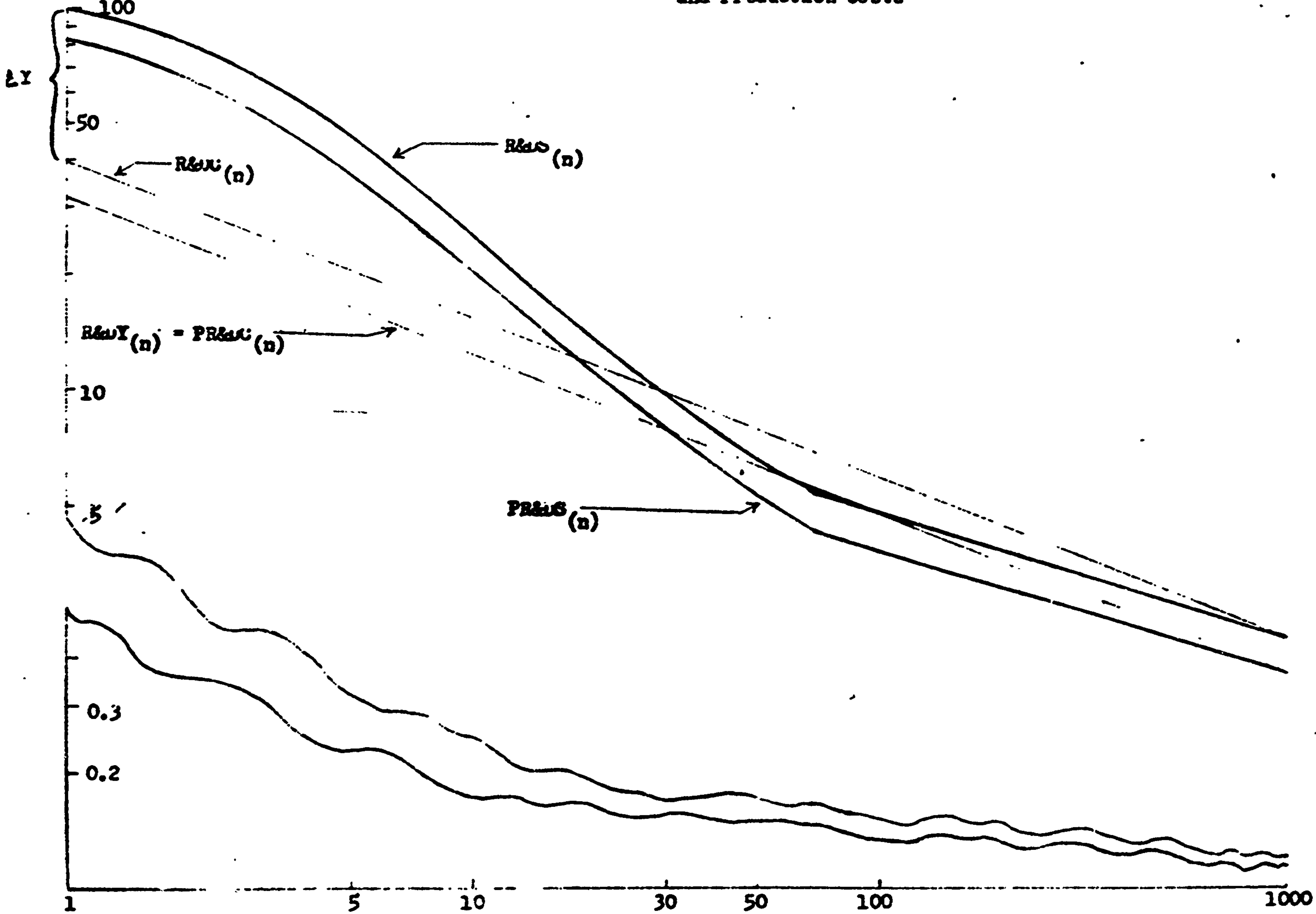
4.1.1 The Ratio or $R\&DS_{(n)}$ to $R\&DC_{(n)}$

The ratio (distance) ΔY, see Figure 3, between $R\&DS_{(1)}$ and $R\&DC_{(1)}$ can be determined by the reciprocal relationship of the slope of $R\&DC_{(n)}$, the expected slope of $P_{(n)}$, and ΔY; based on the premise that the total cost of the S-curve must equal the total cost of the characteristic curve:

$$\frac{1}{((\text{Slope } R\&DC_{(n)} - (\text{Slope } P_{(n)} - \text{Slope } R\&DC_{(n)}))\ \Delta Y} = 1 \quad \text{or,}$$

$$\frac{1}{\text{Slope } R\&DC_{(n)} = (\text{Slope } P_{(n)} - \text{Slope } R\&DC_{(n)})} = \Delta Y$$

Fig. 3. R&D S-Curve for R&D Prototype and Production Costs



For example:

$$R\&DC_{(n)} = 75\%,\ P_{(n)} = 85\%, \text{ then}$$

$$\Delta Y = \frac{1}{.75 = (.85 = .75)} = 1.53846, \text{ i.e.,}$$

the distance between $R\&DS_{(1)}$ and $R\&DC_{(1)}$ is 1.53846 X the unit one value of $R\&DC_{(n)}$.

As $\Delta Y > 1$, it is necessary to determine the values for units 2, 3, 4 and 5 of $R\&DS_{(n)}$. This can be done first as an approximation, followed by a refinement of the approximation.

4.1.2 Determining Units 2, 3, 4 and 5 of $R\&DS_{(n)}$

An approximation of the shape of $R\&DS_{(n)}$, where $2 \leq n \leq 5$, can be made in the subsequent manner:

Step 1 is to place $R\&DS_{(1)}$

Step 2 is to plot $R\&DC_{(n)}$ at a distance ΔY from $R\&DS_{(1)}$

Step 3 is to mark off the value for units $R\&DS_{(10)}$ through $R\&DS_{(1000)}$ per Cochran's concept

Step 4 is to compute the approximate values for units 2, 3, 4 and 5 of $R\&DS_{(n)}$ as:

$$R\&DS_{(n)} = \left[\left(\frac{R\&DS_{(i)}}{R\&DY_{(i)}}\right)\left(\frac{1}{(1+X) - [(i-1)(N-i-1)]}\right) + 1\right] R\&DY_{(i)}$$

where

X = level of experience, i.e., the number of units produced <u>for a given configuration</u>

i = the exponential of the series 2^m which expresses the number of production units necessary to complete the learning cycle of the log-linear curve from m=0 to m=m, or i=m

N = the number of units in learning cycle i

or,

2^m	2^0	2^1	2^2
i	0	1	2
N	1	2	4
n	#1	#2, #3	#4,#5,#6,#7

1 Unit   2 Units   3 Units

$$R\&DY_{(i)} = \frac{(R\&DS_{(i)})(\text{Unit Factor } R\&DS_{(100)})}{\text{Unit Factor } R\&DC_{(100)}}$$

Unit Factor $R\&DC_{(100)}$ is readily available in 2/

Unit Factor $R\&DS_{(100)} = R\&DS_{(100)}/R\&DS_{(1)}$

Step 5 is to plot the approximated values and connect all points with a smooth curve.

Step 6 is to read the refined values of the approximated values from the curve.

The values for units not marked off in Step 3 can be determined by bringing between the known unit values with the Triangular Method[5/] of determining the slopes.

At this point we have constructed the R&D S-curve for R&D prototype or preproduction model costs.

4.2 R&D Production Costs

As in the normal production phase, the R&D production units that are produced are duplicates of the R&D prototype or preproduction models. Hence, where major change has ceased, the unit one cost of the base R&D production curve $PR\&DS_{(n)}$ would be equal to the unit two cost of $R\&DS_{(n)}$, or $PR\&DS_{(1)} = R\&DS_{(2)}$ and $PR\&DS_{(2)} = PR\&DS_{(1)}$ $(R\&DS_{(2)}/R\&DS_{(1)})$,. . ., $PR\&DS_{(n)} = PR\&DS_{(1)}$ $(R\&DS_{(n)}/R\&DS_{(1)})$.

Thus, $PR\&DS_{(n)}$ can be plotted readily as shown in Figure 3, in addition, as $R\&DY_{(n)} = PR\&DC_{(n)}$, $R\&DS_{(2)}$ can be further expressed as $R\&DS_{(2)} = (R\&DY_{(1)}/R\&DC_{(1)})\ R\&DS_{(1)}$.

The total R&D production cost for a given number of units and a given level of experience n-1, where n - learning curve units, is the summation of those units on the log-linear curve (slope = slope of $R\&DC_{(n)}$) with unit one equal to $PR\&DS_{(n)}$.

4.3 Computing the Unit of Experience Factor

Progress or experience in R&D is accomplished in varying degrees with respect to the number of R&D prototype or production units. The state-of-the-art or unit of experience factor $EF_{(n-1)}$ for a given situation can be measured as (see Table 1):

$$EF_{(n-1)} = R\&DS_{(n)}/PR\&DS_{(n+1)}$$

where, n = learning curve unit

The state-of-the-art and, as R&D efforts are made, the degree of progress can be determined from this measure. In addition, an optimum R&D investment point can be projected with this measure for a given configuration.

5. DETERMINING FULL SCALE PREPRODUCTION MODEL $PPS_{(n)}$ AND PRODUCTION UNIT $PS_{(n)}$ COSTS FROM R&D COSTS

In the previous discussion means were developed to determine the cost of the effect of change, the cost of change, and the base production in the R&D phase. In order to determine full scale production costs from R&D costs, a means to determine the ratio of these costs is needed.

From the basic $S_{(n)}$, $C_{(n)}$ parameters we can deduce that:

$$\sum_{n=1}^{n=A} R\&DS_{(n)} = \sum_{n=1}^{n=A} PS_{(n)} + \left(\sum_{n=1}^{n=A} PPS_{(n)} - \sum_{n=1}^{n=A} PS_{(n)} \right) +$$

$$\left(\sum_{n=1}^{n=A} PR\&DS_{(n)} = \sum_{n=1}^{n=A} PPS_{(n)} \right) + \left(\sum_{n=1}^{n=A} R\&DS_{(n)} - \sum_{n=1}^{n=A} PR\&DS_{(n)} \right)$$

Further, it has been shown that $R\&DY_{(n)}$ must intercept $R\&DS_{(n)}$ at $R\&DS_{(100)}$.

Then, it follows that

$$PPS_{(1)} = R\&DC_{(1)}$$

$$PS_{(1)} = R\&DY_{(1)}$$

and the ratio of R&D cost to full scale production cost (dy), see Fig. 4, is equal to ΔY.

Therefore,

$$PPU_{(1)} = R\&DC_{(1)}/dy$$

$$PC_{(1)} = PR\&DC_{(1)}/dy, \text{ where } dy = \Delta Y.$$

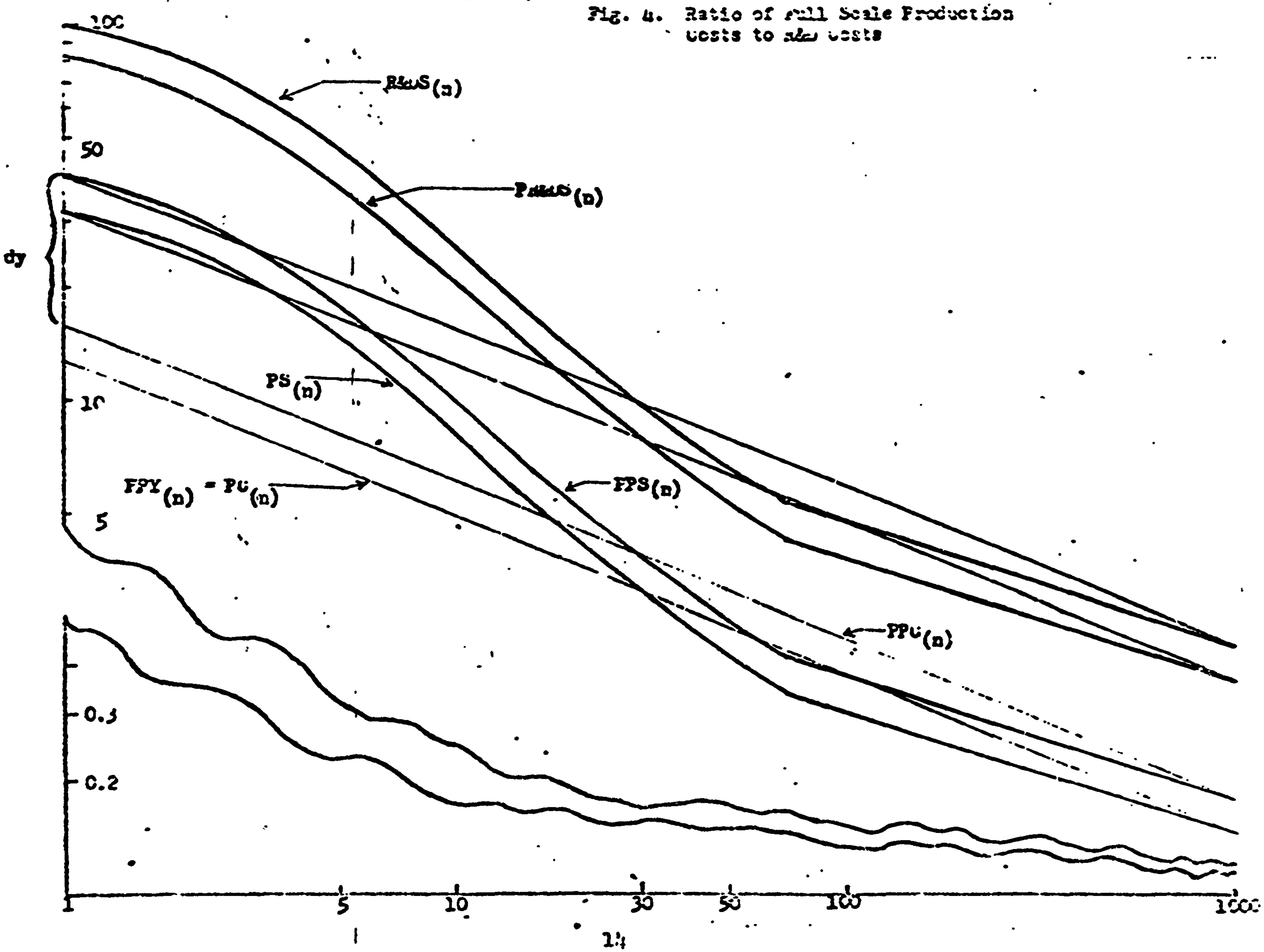
For further proof of the above, it should be noted that $R\&DY_{(n)} = PR\&DC_{(n)}$ and $PPY_{(n)} = PC_{(n)}$.

Thus, full scale production preproduction model cost $PPS_{(n)}$ and unit production cost $PS_{(n)}$ can readily be determined from R&D costs.

6. THE CONSTRUCTION OF R&D/PRODUCTION S-CURVE TABLES

Tables in the general form can readily be constructed for any combination of R&D and production curves as has been done for the basic log-linear curve.[3]

Fig. 4. Ratio of Full Scale Production Costs to R&D Costs



An example of table construction in the general form follows:

Given: Slope of $R\&DC_{(n)}$ = 75%

Expected Production Slope of $P_{(n)}$ = 85%

$R\&DS_{(1)}$ = 100, as this is the general form

(1) $$\Delta Y = \frac{1}{\text{Slope } R\&DC(n) - (\text{Slope } P_{(n)} - \text{Slope } R\&DC_{(n)})}$$

$$\Delta Y = \frac{1}{.75 - (.85 - .75)} = 1.53846$$

(2) $R\&DC_{(1)} = R\&DS_{(1)}/\Delta Y{+}1 = 100/2{,}5385 = 39.39334$

$R\&DC_{(1000)} = R\&DC_{(1)}$ x Unit Factor(75%) for unit 1,000[3/] = 39.39334 x .0568706 = 2.24032

(3) $$R\&DS(70) = \frac{R\&DC_{(1000)} \text{ x Unit Factor } (80\%) \text{ for Unit 70}}{\text{Unit Factor } (80\%) \text{ for Unit 1,000}}$$

$$= (2.24032 \times .2546895)/.1081971 = 5.27358$$

Similarly, $R\&DS_{(100)}$ = 4.70152 and

$R\&DY_{(1)}$ = 31.79170, from $R\&DS_{(100)} = R\&DY(100)$,

$R\&DS(2)$ = 80.70323, from $(R\&DY_{(1)}/R\&DC_{(1)})\ R\&DS_{(1)}$.

(4) $$R\&DS_{(n)} = \left[\left(\frac{S^i\ (1)}{Y^i\ (1)}\right)\left(\frac{1}{(1+x) - (i-1)\ (N-i-1)}\right) + 1\right] Y^i\ (1)$$

$$R\&DS_{(3)} = \left[\left(\frac{100}{31.79159}\right)\left(\frac{1}{(1=2) - [(1-1)\ (2-1-1)]}\right) + 1\right] 31.79159$$

$$= 65.12492$$

Similarly, $R\&DS_{(4)}$ = 53.26677 and $R\&DS_{(5)}$ = 46.45080

(5) Constructing $R\&DS_{(n)}$, $R\&DC_{(n)}$, and $R\&DY_{(n)}$ from the above, (a) plotting points $R\&DS_{(10)}$ through $R\&DS_{(1000)}$ and $R\&DC_{(n)}$ per Cochran's technique, and (b) making a smooth curve through the approximated points $R\&DS_{(3)}$ through $R\&DS_{(5)}$ and the known $R\&DS_{(1)}$, $R\&DS_{(2)}$, and $R\&DS_{(10)}$.

(6) Using the Triangular Method[5], determine the slopes between units $R\&DS_{(2)}$ through $R\&DS_{(70)}$. Refine these slopes by bridging between the known units 2, 10, 15, 20, 30, 40, 50, and 70.

(7) From the refined slopes, determine the remaining unit values from $R\&DS_{(1)}$ through $R\&DS_{(70)}$ (any additional unit values can be determined directly from the Point G line).

(8) As $R\&DS_{(2)} = PR\&DS_{(1)}$, $PPS_{(2)} = PS_{(1)}$, and $R\&DS_{(1)} = 100$:

$PR\&DS_{(2)} = PR\&DS_{(1)}(R\&DS_{(2)}/100)$, $PR\&DS_{(3)} = PR\&DS_{(1)}(R\&DS_{(3)}/100), \ldots,$

$PR\&DS_{(70)} = PR\&DS_{(1)}(R\&DS_{(70)}/100)$.

Similarly, $PPS_{(2)} = PPS_{(1)}(R\&DS_{(2)}/100), \ldots,$ and

$$PS_{(2)} = PS_{(1)}(R\&DS_{(2)}/100), \ldots \quad .$$

(9) Computing the R&D unit of experience factors $EF_{(n-1)}$:

$$EF_{(0)} = R\&DS_{(1)}/PR\&DS_{(2)}, EF_{(1)} = PR\&DS_{(2)}/PR\&DS_{(3)}, \ldots ,$$

$$EF_{(n-1)} = R\&DS_{(n)}/PR\&DS_{(n+1)}$$

(10) Constructing the above into a table yields the subsequent table 1.

Table 1

R&D = 75%, P = 85%, SN = 80%

UNIT OF EXPERIENCE	LEARNING CURVE UNIT	$R\&DS_{(n)}$	$PR\&DS_{(n)}$	$PPS_{(n)}$	$PS_{(n)}$	$EF_{(n-1)} = R\&DS_{(n)}/PR\&DS_{(n+1)}$	SLOPE (%)
Ø	1	100.000	80.7032	39.3933	31.7917	1.53539	Units 1-2 = 80.7032
1	2	80.7032	65.1301	31.7917	25.6569	1.53300	Units 2-3 = 69.5000
2	3	65.2316	52.6440	25.6969	20.7382	1.48358	Units 3-4 = 64.8000
3	4	54.4822	43.9689	21.4624	17.3208	1.45903	Units 4-5 = 60.2000
4	5	46.2701	37.3415	18.2274	14.7101	1.44598	Units 5-7 = 55.6000
5	6	39.6503	31.9991	15.6196	12.6055	1.41190	
6	7	34.7979	28.0830	13.7081	11.0628	1.40306	Units 7-15 = 52.4637
7	8	30.7316	24.8014	12.1062	9.7701	1.38265	
8	9	27.5412	22.2266	10.8494	8.7558	1.36676	
9	10	24.9689	20.1507	9.8361	7.9380	1.35403	
10	11	22.8496	18.4404	9.0012	7.2643	1.34361	
11	12	21.0724	17.0061	8.3011	6.6993	1.33494	
12	13	19.5597	15.7853	7.7052	6.2184	1.32756	
13	14	18.2563	14.7334	7.1918	5.8040	1.32128	
14	15	17.1210	13.8171	6.7445	5.4431	1.30871	Units 15-20 = 55.6000
15	16	16.2104	13.0823	6.3858	5.1536	1.30439	
16	17	15.3991	12.4276	6.0662	4.8956	1.30056	
17	18	14.6715	11.8404	5.7796	4.6643	1.29716	
18	19	14.0149	11.3105	5.5209	4.4556	1.29412	
19	20	13.4192	10.8297	5.2863	4.2662	1.29003	Units 20-30 = 56.4284
20	21	12.8895	10.4022	5.0776	4.0978	1.28762	
21	22	12.4039	10.0103	4.8863	3.9434	1.28543	
22	23	11.9569	9.6496	4.7102	3.8013	1.28341	
23	24	11.5441	9.3165	4.5476	3.6701	1.28157	
24	25	11.1616	9.0078	4.3969	3.5485	1.27988	

UNIT OF EXPERIENCE	LEARNING CURVE UNIT	$R\&DS_{(n)}$	$PR\&DS_{(n)}$	$PPS_{(n)}$	$PS_{(n)}$	$EF_{(n-1)} = R\&DS_{(n)}/PR\&DS_{(n+1)}$	SLOPE (%)
25	26	10.8060	8.7208	4.2568	3.4354	1.27832	
26	27	10.4745	8.4533	4.1263	3.3300	1.27686	
27	28	10.1648	8.2033	4.0043	3.2316	1.27554	
28	29	9.8745	7.9690	3.8899	3.1393	1.27428	
29	30	9.6020	7.7491	3.7825	3.0526	1.27022	Units 30-40 = 59.2000
30	31	9.3668	7.5593	3.6899	2.9779	1.26921	
31	32	9.1446	7.3800	3.6024	2.9072	1.26829	
32	33	8.9342	7.2102	3.5195	2.8403	1.26741	
33	34	8.7347	7.0492	3.4409	2.7769	1.26658	
34	35	8.5453	6.8963	3.3663	2.7167	1.26578	
35	36	8.3652	6.7510	3.2953	2.6594	1.26506	
36	37	8.1936	6.6125	3.2277	2.6049	1.26435	
37	38	8.0300	6.4805	3.1633	2.5529	1.26369	
38	39	7.8738	6.3544	3.1018	2.5032	1.26308	
39	40	7.7244	6.2338	3.0429	2.4557	1.26127	Units 40-60 = 60.8000
40	41	7.5887	6.1243	2.9894	2.4126	1.26073	
41	42	7.4586	6.0193	2.9382	2.3712	1.26022	
42	43	7.3337	5.9185	2.8890	2.3315	1.25974	
43	44	7.2136	5.8216	2.8417	2.2933	1.25925	
44	45	7.0982	5.7285	2.7962	2.2566	1.25881	
45	46	6.9871	5.6388	2.7525	2.2213	1.25839	
46	47	6.8800	5.5524	2.7103	2.1873	1.25798	
47	48	6.7768	5.4691	2.6696	2.1545	1.25757	
48	49	6.6773	5.3888	2.6304	2.1228	1.25721	
49	50	6.5811	5.3112	2.5925	2.0922	1.25685	

UNIT OF EXPERIENCE	LEARNING CURVE UNIT	$R\&DS_{(n)}$	$PR\&DS_{(n)}$	$PPS_{(n)}$	$PS_{(n)}$	$EF_{(n-1)} = R\&DS_{(n)}/PR\&DS_{(n+1)}$	SLOPE (%)
50	51	6.4882	5.2362	2.5559	2.0627	1.25650	
51	52	6.3984	5.1627	2.5205	2.0342	1.25616	
52	53	6.3115	5.0936	2.4863	2.0065	1.25584	
53	54	6.2274	5.0257	2.4532	1.9798	1.25584	
54	55	6.1459	4.9599	2.4211	1.9539	1.25555	
55	56	6.0669	4.8962	2.3900	1.9288	1.25494	
56	57	5.9903	4.8344	2.3598	1.9044	1.25467	
57	58	5.9160	4.7744	2.3305	1.8808	1.25443	
58	59	5.8438	4.7161	2.3021	1.8578	1.25414	
59	60	5.7738	4.6596	2.2745	1.8356	1.25123	Units 60-70 = 66.5362
60	61	5.7179	4.6145	2.2525	1.8178	1.25102	
61	62	5.6635	4.5706	2.2310	1.8005	1.25080	
62	63	5.6105	4.5279	2.2102	1.7837	1.25064	
63	64	5.5588	4.4861	2.1898	1.7672	1.25043	
64	65	5.5084	4.4455	2.1699	1.7512	1.25029	
65	66	5.4592	4.4057	2.1506	1.7356	1.25013	
66	67	5.4111	4.3669	2.1316	1.7203	1.24994	
67	68	5.3642	4.3291	2.1131	1.7054	1.24978	
68	69	5.3184	4.2921	2.0951	1.6908	1.24962	
69	70	5.2736	4.2560	2.0774	1.6766	1.24477	Units 70-1,000=80.000
99	100	4.7015	3.7943	1.8521	1.4947	1.24308	
499	500	2.8004	2.2600	1.1032	0.8903	1.23991	
999	1000	2.2403	1.8080	0.8825	0.7122	1.23949	

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